the implicit step is an efficient and rapid way of obtaining reasonably accurate solutions.

## References

[1]Tong, S. S. and Thompkins, W. T., "A Design Calculation Procedure for Shock-Free or Strong Passage Shock Turbomachinery Cascades," *Journal of Engineering for Power*, Vol. 105, No. 2, April 1983, pp. 369-376.

[2]Thompkins, W. T., Tong, S. S., Bush, R. H., Usab, W. J., and Norton, R. J. G., "Simulations of Flow in Turbomachinery Cascades," AIAA Paper 83-0257, Jan. 1983.

[3]Thompkins, W. T. and Tong, S. S., "Inverse or Design Calculations for Non-Potential Flow in Turbomachinery Blade Passages," ASME Paper 81-GT-78, 1981.

[4]Steger, J. L., Pulliam, T. H., and Chima, R. V., "An Implicit Finite Difference Code for Inviscid and Viscous Cascade Flow," AIAA Paper 80-1427, July 1980.

[5]Kordulla, W. and MacCormack, R. W., "Transonic Flow Computation Using an Explicit-Implicit Method," *Proceedings of the Eighth International Conference on Numerical Methods in Fluid Dynamics*, Springer-Verlag, Berlin, 1982, pp. 286-295.

[6]Beam, R. and Warming, R. F., "An Implicit Factored Scheme for the Compressible Navier-Stokes Equation," AIAA Paper 77-645, June 1977.

[7]Steger, J. L., "Implicit Finite Difference Simulation of Flow About Arbitrary Two Dimensional Geometries," *AIAA Journal*, Vol. 16, July 1978, pp. 679-686.

[8]von Lavante, E. and Iyer, V., "Simplified Implicit Block-Bidiagonal Finite Difference Method for Soving the Navier-Stokes Equations," *AIAA Journal*, Vol. 23, July 1985, pp. 1130-1132.

[9]Iyer, V., "Numerical Solutions of Compressible Viscous Flow in Transonic Compressor Cascades," Ph.D. Dissertation, Dept. of Aerospace Engineering, Texas A&M University, College Station, May 1985.

[10]Iyer, V. and von Lavante, E., "Numerical Solutions of Viscous Transonic Flow in Turbomachinery Cascades," AIAA Paper 85-0007, Jan. 1985.

[11]Bryanston-Cross, P. J. and Denton, J. D., "Comparison of Interferometric Measurements and Computed Flow Around a Wedge Profile in the Transonic Region," ASME Paper 82-GT-258.

# Optimal Barrel-Shaped Shells Under Buckling Constraints

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## I. Introduction

DOUBLY curved shells are common structural elements in various industries (aerospace, marine, civil engineering, etc.), and the buckling of such elements has received much attention in design.[1-6] It is generally agreed that the papers by McElman and Stein[1,2] sparked off detailed analysis of barreled shells. Their theory was limited to shallow shells and to buckling near the equator of a toroidal shell segment. Solutions were obtained, in a closed analytical form, for simply supported shells of both positive and negative curvature under internal or external pressure. A finite-element solution, as well as some experimental results presented in Ref. 7, did not completely agree with these first solutions.

The buckling and postbuckling behavior of axially compressed bowed-out and bowed-in shells was discussed in Ref. 8. Linear shell theory was used to find the buckling load, whereas the nonlinear theory was adopted to investigate the postbuckling state.

Qualitative ways of increasing the buckling resistance of barreled shells via meridional curvature change, stiffener eccentricity, and load eccentricity have been pointed out in Refs. 9-14. A substantial increase in the resistance of these shells to buckling has been observed, although these studies have not been based on optimal configurations.

The minimum weight of stiffened and barreled shells was examined in Ref. 15. In order to obtain the minimum weight configuration for a fixed axial load, the thickness of the skin and the thickness, depth, and spacing of solid rectangular stiffening rings and stringers were sought. Buckling was based on the linear membrane prebuckling state, and the gradient method with a penalty function was used.

The main aim of this paper is to establish, parametrically, the maximal compressive axial buckling load of a bowed-out shell. The wall thickness and the shell volume are kept constant. The buckling load increase is due to the change in the shell meridional curvature.

## II. Formulation of Problem

Consider an elastic, clamped-clamped cylindrical shell of radius $R_0$, length $L_0$, constant thickness $t$, under axial compression $F$ (see Fig. 1a). For a given volume of the cylindrical shell $V_{\text{cyl}}$, we seek the meridional radius of curvature $r$ of the toroidal shell (see Fig. 1b) which possesses the same volume, thickness, boundary conditions, and truncation radius $R_0$, and also maximizes the buckling load.

## III. Method of Solution

The volume of the toroidal shell (Fig. 1b) can be written as

$$V_{\text{tor}} = 4\pi(r^2 \sin\alpha - br\alpha)t \tag{1}$$

For a given value of the distance $b$, from the axis of revolution to the center of meridional curvature (see Fig. 1b), one can find the radius $r$ from

$$V_{\text{tor}} = V_{\text{cyl}} \tag{2}$$

or

$$2(r^2 \sin\alpha - br\alpha) = R_0 L_0 \tag{3}$$

where $R_0, L_0$ are supposed to be given.

For a given value of the control parameter $b$ we obtain, via Eq. (3), all the necessary geometrical parameters of the toroidal shell, which has the same volume as the original cylindrical shell. The computer code BOSOR 4[16] is used here to find the bifurcation buckling load and the collapse load. The latter is found by tracing out the load-displacement curve beyond the limit point (applying displacement-controlled analysis). Varying $b$ in the range from zero to infinity, one obtains the truncated sphere and cylinder, respectively, as limiting cases.

## IV. Numerical Results

Calculations were performed for $L_0/R_0$ varying from 0.25 to 4.0 and $R_0/t$ varying from 100 to 1000. Figure 2 shows the change of bifurcation load as a function of curvature. For $L_0/R_0 = 4$, the optimal load is obtained for $R_0/(R_0+b) = 0.09$, regardless of the parameter $R_0/t$ within the range of 100-1000. Analogous values for $L_0/R_0 = 2$ and $L_0/R_0 = 1$ are 0.17 and 0.28, respectively. Additional results for $L_0/R_0 = 0.5$ and $L_0/R_0 = 0.25$ are depicted in Fig. 3. It is seen that the optimal configurations do not depend on $R_0/t$ for $0.5 \leq L_0/R_0 \leq 4.0$. However, for $L_0/R_0 = 0.25$ and $R_0/t < 300$, a change in meridional curvature no longer increases the buckling load. Axisymmetric collapse now be-

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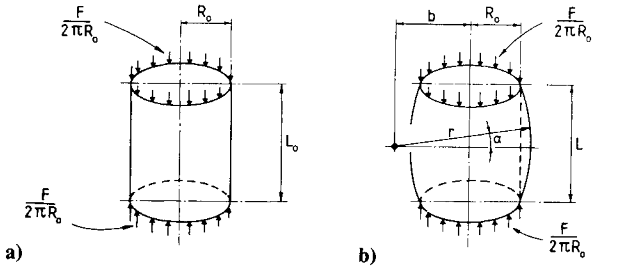


Fig. 1 Geometry of an a) cylindrical and b) barreled-out shell.

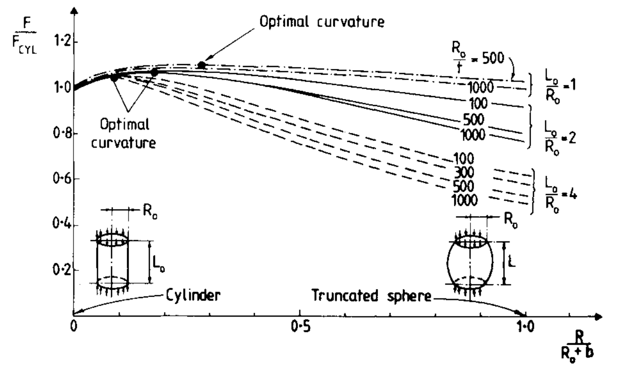


Fig. 2 Bifurcation load vs control variable $R_0/(R_0+b)$ for $L_0/R_0=1$, 2, and 4.

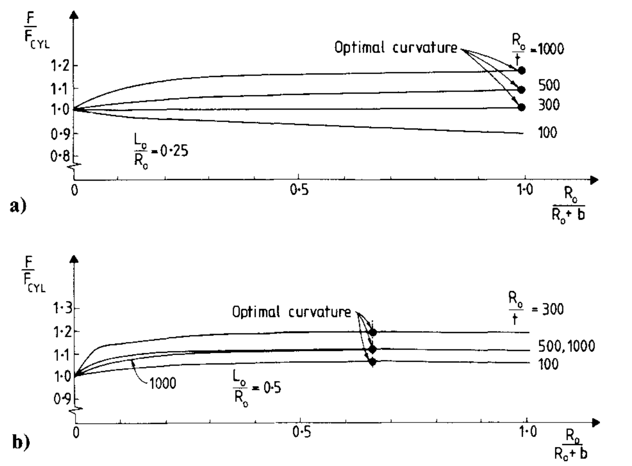


Fig. 3 Bifurcation load vs control variable $R_0/(R_0+b)$ for a) $L_0/R_0=0.25$ and b) $L_0/R_0=0.5$.

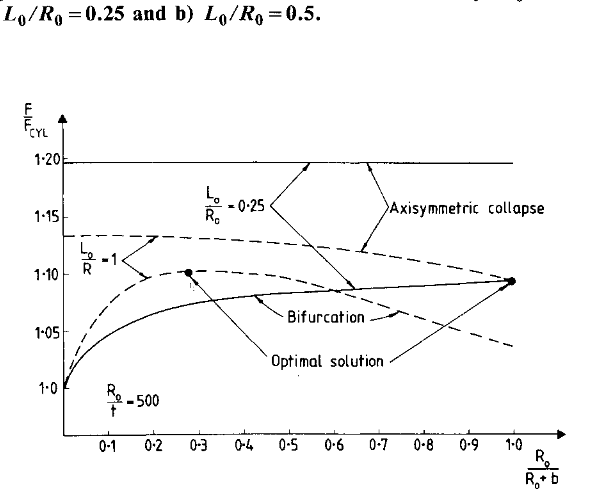


Fig. 4 Bifurcation and axisymmetric collapse loads vs amount of barreling out for $L_0/R_0=1$ and $L_0/R_0=0.25$.

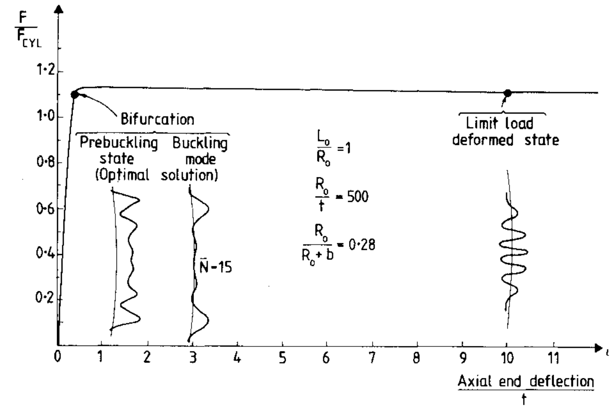


Fig. 5 Load-deflection behavior of an optimally bowed-out shell.

comes the controlling mode, and the collapse load decreases as the amount of barreling out increases (see Fig. 3a). Limit and bifurcation loads are displayed together in Fig. 4, for some of the parameters investigated heretofore. The load-displacement curve is shown for the optimal value of the control variable $R_0/(R_0+b)=0.28$ in Fig. 5. The bifurcation and limit loads are denoted on this curve. The prebuckling state and buckling mode are added as inserts for the bifurcation load; the deformed state is shown for the limit load.

## V. Conclusions

As a result of this study, the following observations can be made:

1) The controlling mode of failure is bifurcation buckling for $0.5 \leq L_0/R_0 \leq 4$, and the optimal critical load does not depend on $R_0/t$. For shorter cylinders (e.g., $L_0/R_0=0.25$), the controlling mode is either bifurcation buckling ($R_0/t \geq 300$) or axisymmetric collapse ($R_0/t<300$).

2) There is no dramatic increase in the buckling load, although gains of up to 20% can be achieved via the curvature shaping.

3) Barreling out does not necessarily lead to an increase in buckling load, as is commonly assumed in the literature.

4) There is a wide plateau of curvature values where a small change in the curvature distribution does not cause a big change in the load-carrying capacity for $0.25 \leq L_0 \div R_0 \leq 1.0$.

## References


[1]Stein, M. and McElman, J. A., "Buckling of Segments of Toroidal Shells," *AIAA Journal*, Vol. 3, Sept. 1965, pp. 1704–1709.

[2]McElman, J. A., "Eccentrically Stiffened Shallow Shells of Double Curvature," NASA TN D-3826, 1967.

[3]Odland, J., "Theoretical and Experimental Buckling Loads of Imperfect Spherical Shell Segments," *Journal of Ship Research*, Vol. 25, No. 3, 1981, pp. 201–218.

[4]Wurm, P., "Modellversuche zum Studium des Beulverhaltens doppelt positiv gekrümmter Schalen," *Der Bauingenieur*, Vol. 50, Oct. 1975, pp. 397–403.

[5]Zerna, W. and Mungan, I., "Buckling of Shells having Negative Gaussian Curvature," *Buckling of Shells, Proceedings of a State-of-the-Art Colloquium*, Springer-Verlag, Stuttgart, 1982, pp. 467–485.

[6]Zintilis, G. M. and Croll, J. G. A., "Combined Axial and Pressure Buckling of End Supported Shells of Revolution," *Engineering Structures*, Vol. 5, July 1983, pp. 199–206.

[7]Weingarten, V. I., Veronda, D. R., and Saghera, S. S., "Buckling of Segments of Toroidal Shells," *AIAA Journal*, Vol. 11, Oct. 1973, pp. 1422–1424.

[8]Lukasiewicz, S. and Szyszkowski, W., "On the Stability and the Postbuckling Equilibrium of Shells of Revolution," *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 51, 1971, pp. 635–639.

[9]Almroth, B. O. and Bushnell, D., "Computer Analysis of Various Shells of Revolution," *AIAA Journal*, Vol. 6, Oct. 1968, pp. 1848–1855.

[10]Becker, H., "Compressive Buckling of Barrel-Shaped Shells," *Journal of Spacecraft and Rockets*, Vol. 5, June 1968, pp. 746-747.

[11]Blum, R. E. and McComb, H. G., "Buckling of an Equatorial Segment of a Spherical Shell Loaded by its Own Weight," NASA TN D-4921, 1968.

[12]Stein, M., "Some Recent Advances in the Investigation of Shell Buckling," *AIAA Journal*, Vol. 6, Dec. 1968, pp. 2339-2345.

[13]Bushnell, D., "Stress, Stability and Vibration of Complex, Branched Shells of Revolution," *Computers and Structures*, Vol. 4, 1974, pp. 399-435.

[14]Hutchinson, J. W. and Frauenthal, J. C., "Elastic Postbuckling Behaviour of Stiffened and Barrelled Cylindrical Shells," *Journal of Applied Mechanics*, ASME, Vol. 36, Dec. 1969, pp. 784-790.

[15]Stroud, W. J. and Sykes, N. P., "Minimum Weight Stiffened Shells with Slight Meridional Curvature Designed to Support Axial Compressive Loads," *AIAA Journal*, Vol. 7, Aug. 1969, pp. 1599-1601.

[16]Bushnell, D., "BOSOR 4 Program for Stress, Buckling, and Vibration of Complex Shells of Revolutions," *Structural Mechanics Software Series*, University Press of Virginia, Charlottesville, VA, 1977, pp. 11-143.